Finding areas under the normal

A linear function of a normal random variable is also normally distributed. In general if (as in p. 24):

$$x = \mu + \epsilon$$
 $\epsilon \sim N(0, \sigma^2)$

then,

$$x - \mu = \epsilon \qquad \Rightarrow x - \mu \sim N(0, \sigma^2)$$

and finally,

$$\frac{x-\mu}{\sigma} = \frac{\epsilon}{\sigma} \qquad \Rightarrow \frac{x-\mu}{\sigma} \sim N(0,1)$$

which represents deviations from mean in units of standard deviation.

Example

Suppose X is normally distributed with mean 10, and variance 25, that is,

 $X \sim N(10, 25)$

Then, what is the probability

$$P(12 \le X \le 15) =?$$

Given the mean and the variance Excel calculates this probability using the command normdist(X,mean,st dev,1). In this case the whole command will be:

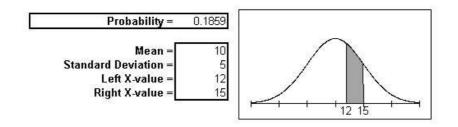
Kadd (the Excel add-in) has a **Probability** command that calculates this probability by specifying the mean, the standard deviation, and the interval (12-15).

You can verify this using a table for the stardard normal and calculating the following probability:

$$P(12 \le X \le 15) = P\left(\frac{12-\mu}{\sigma} \le \frac{X-\mu}{\sigma} \le \frac{15-\mu}{\sigma}\right)$$

= $P\left(\frac{12-10}{5} \le Z \le \frac{15-10}{5}\right)$
= $P(0.4 \le Z \le 1)$
= $P(0 \le Z \le 1) - P(0 \le Z \le 0.4)$
= $0.3413 - 0.1554$
= 0.1859

which graphically can be represented as follows:



Area under the Normal Distribution